Lyapunov Functions old and new

a two weeks intensive course by Albert Fathi (École Normale Supérieure de Lyon)

The importance of Lyapunov functions for stability of attractors in Dynamical Systems goes back to Lyapunov. Converse Lyapunov results have been an important contribution of Massera.

For attractors smoothness of the Lyapunov function has not been an issue. Charles Conley has generalized the theory to include isolated blocks. There saddle type behaviour is predominant and smoothness of the Lyapunov function is an issue.

There is a long history of false claims concerning smoothness of Lyapunov functions. Ethan Akin settled the matter for homeomorphisms, however the situation for flows is not settled in the literature. Lyapunov functions on Aubry sets appear naturally as differences of subsolutions of Hamilton-Jacobi equation. Usually they are not smooth enough. This has recently renewed interest in the deeper understanding of Lyapunov functions.

The course is an introduction to a new circle of ideas in Lyapunov function theory that originated in Aubry-Mather theory. It will be elementary and self-contained. Here is a description of the topics:

1) Lyapunov functions and attractors.
2) Massera's converse Lyapunov for attractors and smoothness.
3) Chain recurrence and smoothness of Lyapunov functions. Examples.
4) Constructions of Lyapunov functions via a Conley barrier.
5) How to smooth Lyapunov function for a homeomorphism or differentiable vector field. Counterexamples when the flow is just continuous.
6) A general Aubry-Mather theory for homeomorphisms: Aubry set as Auslander prolongation, and Mañé set as the chain recurrence set.
7) Further directions: time functions in relativity, and utility functions in economics.

References:

Books
Charles Conley, Isolated invariant sets and the Morse index, CBMS Regional Conference Series in Mathematics, 38.
Michael Shub, Global stability of dynamical systems Springer.
Ethan Akin, The General Topology of Dynamical Systems AMS.

Articles


Sullivan D., Cycles for the Dynamical Study of Foliated Manifolds and Complex Manifolds, Inv Math 36 225-255.


DECEMBER 2010 – MONTEVIDEO – URUGUAY (Dates to be confirmed)

More information, financial support, etc: email to emaderna@cmat.edu.uy