Lectures on scattering theory in partial-wave amplitudes

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Brief introduction

We cover some basic topics on scattering theory, emphasizing unitarity and analyticity of the S matrix while keeping an eye on phenomenology. These lectures could be useful to those interested in scattering processes that cannot be calculated by using only perturbative techniques in Quantum Mechanics or in Quantum Field Theory. The different concepts here are introduced from a nonperturbative point of view. The most formal part of the lectures concern the first two chapters, which are the longest ones. The first one is dedicated to formal topics in the theory of the S-matrix, while the second chapter develops in detail the expansion in partial-wave amplitudes of scattering amplitudes and related aspects, like unitarity in partial waves. The third chapter is an introduction to crossing symmetry and its applications. The other chapters are typically of narrower scope, typically more specific, and treat several interesting topics in partial-wave amplitudes: The N/D method, CDD poles, unphysical Riemann sheets and resonance poles there, their nature, and final-state interactions. There is an added appendix dedicated to the relatively unknown Sugawara-Kanazawa theorem, which is applicable to many situations of interest in phenomenology. There is a bunch of exercises along the manuscript typically asking the reader to complete some derivations or deduce new results.

These lectures are rather self-contained and could be used with only limited background knowledge on the subject. Books like [1–6] have been often consulted by the author along his research trajectory and they have imprinted these lectures. In preparing them I have also used my previous book [7]. A large part of this material was presented in a short course given at the Instituto de Física de la Facultad de Ingeniería de la Universidad de la República Uruguay.